COMPARISON OF THEORETICAL PREDICTIONS OF RADIATION EFFICIENCY WITH EXPERIMENTAL MEASUREMENTS

John Laurence Davy

School of Applied Sciences, RMIT University, GPO Box 2476, Melbourne, Victoria 3001, Australia
e-mail: johnldavy@gmail.com

Jeffrey Mahn

National Research Council of Canada, 1200 Montreal Rd, Ottawa ON K1A 0R6, Canada

Luboš Krajči

Soundtherm, Acoustics and Physics of the Buildings, Bratle 9, 8255 Schlattingen, Switzerland

Robin Wareing and John Pearse

University of Canterbury, Mechanical Engineering, Private Bag 4800, Christchurch 8140, New Zealand

The measurement and prediction of radiation efficiency is important in many areas of acoustics. It has recently become an even more important topic, because the radiation efficiency needs to be known in order to predict the flanking sound transmission using EN12354 when the frequency is below the critical frequency of one or more of the walls involved in the flanking sound transmission path. This prediction is more difficult for lightweight walls because they usually consist of a complex construction. This paper uses Maidanik's and Leppington's approximate formulae to predict the resonant radiation efficiency of a finite size panel and Davy's approximate formulae to predict the forced radiation efficiency when a finite size panel is excited by a diffuse sound field. The radiation efficiency of an infinite size panel excited by a point or line force is predicted using Heckl's approximate formulae. The damping loss factor of the panel is needed in order to calculate the magnitude of the resonant vibration of the panel relative to the point or line near field vibration of the panel or relative to the vibration of the panel forced by an incident diffuse sound field on one side of the panel. The theories used in this paper assume that the finite size panel is mounted in an infinite baffle and the predictions are for the power radiated on one side of the panel. This paper compares the theoretical predictions of radiation efficiency with experimental measurements made in sound insulation laboratories. The surface velocities of the panels were measured with a laser velocimeter or accelerometers. The sound power radiated on one side of the panel was determined using sound intensity measurements or reverberation room measurements of sound power that utilised sound pressure measurements and reverberation time measurements.

1. Introduction

The radiation efficiency of a wall needs to be known in order to make many predictions in acoustics. For example, the attempt to extend the EN 12354 flanking sound transmission standards [1, 2] to frequencies below the critical frequency of the walls involved has highlighted this need to know the radiation efficiencies of the walls. This paper presents approximate formulae for the radiation efficiencies of rectangular panels and compares the predictions of these formulae with experimental measurements.

2. The prediction of the radiation efficiency of a rectangular panel

This section gives approximate formulae for the single sided radiation efficiencies of a simply supported rectangular panel mounted in an infinite rigid baffle. Define three empirical constants [3].

$$n = 2 \quad (1)$$

$$w = 1.3 \quad (2)$$

$$\beta = 0.124 \quad (3)$$

Define the length of the side of an equivalent square panel as [3]:

$$2a = \frac{4S}{U} \quad (4)$$

where S is the area of the panel and U is the perimeter of the rectangular panel.

Using the wave number:

$$k = \frac{2\pi f}{c} \quad (5)$$

where c is the speed of sound in air and f is the frequency, calculate some intermediate values [3]:

$$p = \begin{cases} w\sqrt{\dfrac{\pi}{2ka}} & \text{if } w\sqrt{\dfrac{\pi}{2ka}} \le 1 \\ 1 & \text{if } w\sqrt{\dfrac{\pi}{2ka}} > 1 \end{cases} \quad (6)$$

$$h = \frac{1}{\dfrac{2}{3}\sqrt{\dfrac{2ka}{\pi}} - \beta} \quad (7)$$

$$\alpha = \frac{h}{p} - 1 \quad (8)$$

$$q = \frac{2\pi}{k^2 S} \quad (9)$$

Calculate the non-resonant diffuse field excited radiation efficiency [3].

$$\sigma_{nr} = \ln\left(\frac{1+\sqrt{1+q^2}}{p+\sqrt{p^2+q^2}}\right) + \frac{1}{\alpha}\ln\left(\frac{h+\sqrt{h^2+q^2}}{p+\sqrt{p^2+q^2}}\right) \quad (10)$$

Calculate some more intermediate values [3]:

$$t = \frac{f}{f_c} \quad (11)$$

$$g = \begin{cases} \sqrt{1-\dfrac{1}{t}} & \text{if } f \ge f_c \\ 0 & \text{if } f < f_c \end{cases} \quad (12)$$

where f_c is the critical frequency of the panel [3].

$$\sigma_2 = \begin{cases} \dfrac{1}{\sqrt[n]{g^n + q^n}} & \text{if } 1 \geq g \geq p \\ \dfrac{1}{\sqrt[n]{(h - \alpha g)^n + q^n}} & \text{if } p > g \geq 0 \end{cases} \tag{13}$$

If $f < f_c$, calculate σ_1 [4].

$$\sigma_1 = \frac{c}{2a\pi^2 f_c} \frac{[(1-t)\ln\left((1+\sqrt{t})/(1-\sqrt{t})\right) + 2\sqrt{t}]}{(1-t)^{3/2}} \tag{14}$$

Note that Maidanik's [5] extra low frequency term has not been used.

Calculate the resonant radiation efficiency.

$$\sigma_r = \begin{cases} \min(\sigma_1, \sigma_2) & \text{if } f < f_c \\ \sigma_2 & \text{if } f \geq f_c \end{cases} \tag{15}$$

Calculate another intermediate value

$$r = \frac{\pi \sigma_r}{4t\eta} \tag{16}$$

where η is the total in situ damping loss factor. Note that r is the ratio of the resonant vibrational energy to the non-resonant vibrational energy level of a panel which has been excited by a diffuse sound field [6]. The variable r is also the ratio of the power radiated by the resonant vibrational fields to the power radiated by the vibrational near fields for a panel excited by point forces acting at right angles to the panel [7].

Calculate the airborne diffuse field excited radiation efficiency (see equation (11) of [8]).

$$\sigma_a = \begin{cases} \dfrac{r\sigma_r + \sigma_{nr}}{r+1} & \text{if } f < f_c \\ \sigma_r & \text{if } f \geq f_c \end{cases} \tag{17}$$

Calculate the radiation efficiency of a panel excited by point forces acting at right angles to the panel (see equation (28) of [8]).

$$\sigma_p = \begin{cases} \sigma_r \left(1 + \dfrac{1}{r}\right) & \text{if } f < f_c \\ \sigma_r & \text{if } f \geq f_c \end{cases} \tag{18}$$

Calculate the ratio of the power radiated by the resonant vibrational fields to the power radiated by the vibrational near fields for a panel excited by line forces acting at right angles to the panel [7].

$$r_l = \frac{\sigma_r}{2\eta} \sqrt{\frac{f_c}{f}} \tag{19}$$

Calculate the radiation efficiency of a panel excited by line forces acting at right angles to the panel.

$$\sigma_l = \begin{cases} \sigma_r \left(1 + \dfrac{1}{r_l}\right) & \text{if } f < f_c \\ \sigma_r & \text{if } f \geq f_c \end{cases} \tag{20}$$

3. Experimental comparison

The radiation efficiencies were measured for panel of size 1.546 by 0.95 m mounted between a reverberation room and a semi-anechoic room. The average radiated sound intensity was measured on the semi-anechoic room side of the panel. The average velocity of the panel was measured with accelerometers. The panels were excited by a diffuse sound field in the reverberation room to measure σ_a. To measure σ_r, another similar panel was attached at right angles to the original panel in the reverberation room and this panel was excited by a shaker. The damping loss factors were measured using the structural reverberation time [9].

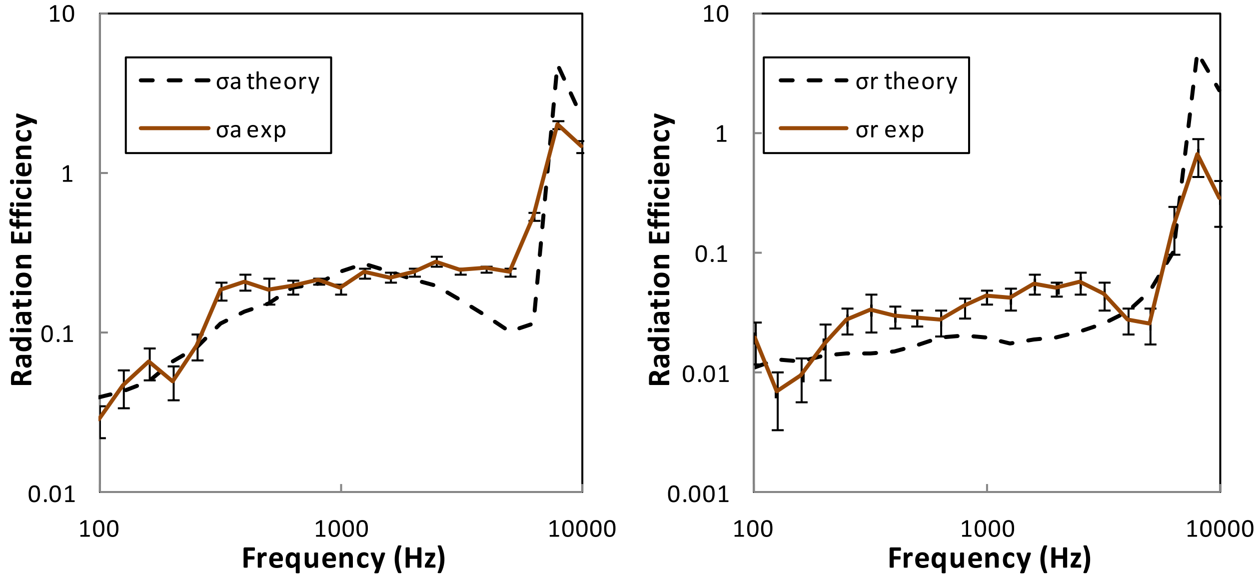

Figure 1. Comparison of the theoretical and experimental diffuse field excited (σ_a) and resonant radiation (σ_r) efficiencies for a 1.6 mm thick steel panel measuring 1.546 by 0.95 m. The experimental 95% confidence limits are shown.

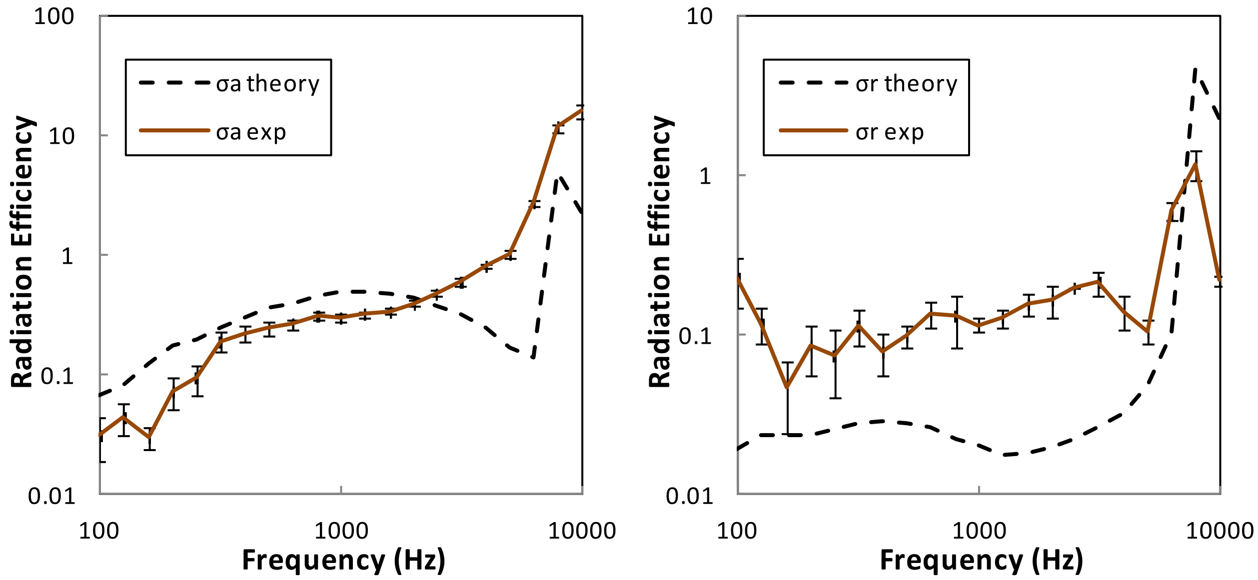

Figure 2. Comparison of the theoretical and experimental diffuse field excited (σ_a) and resonant radiation (σ_r) efficiencies for a 4 mm thick medium density fibre board panel measuring 1.546 by 0.95 m. The experimental 95% confidence limits are shown.

Figures 1 to 3 compare the predicted and measured values for a 1.6 mm thick steel panel, 4 mm medium density fibre board panel and a 10 mm gypsum plaster board panel, respectively. Only σ_a was measured in the case of the gypsum plaster board panel. The predicted values are usually outside the 95% confidence limits, but there is very rough agreement except in the case of σ_r for the medium density fibre board panel. These panels all had relatively high critical frequencies.

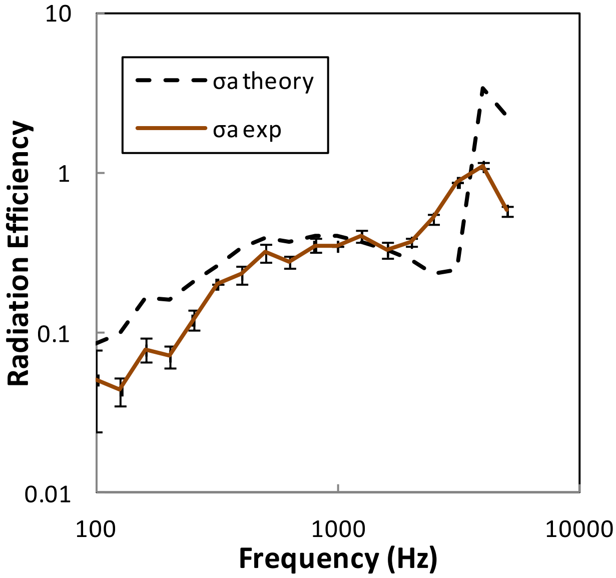

Figure 3. Comparison of the theoretical and experimental diffuse field excited (σ_a) efficiencies for a 10 mm thick gypsum plasterboard panel measuring 1.546 by 0.95 m. The experimental 95% confidence limits are shown.

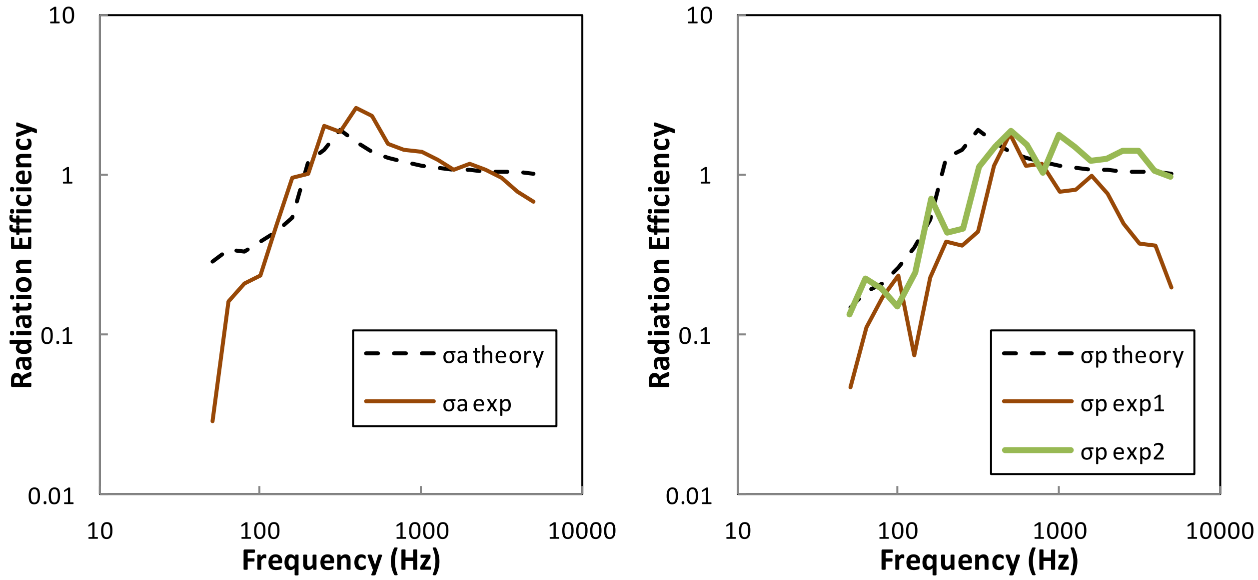

Figure 4. Comparison of the theoretical and experimental diffuse field excited (σ_a) and point excited radiation (σ_p) efficiencies for an 80 mm cross timber laminated panel measuring 4.18 by 2.89 m. There are two experimental measurements of the point excited radiation efficiency.

Figure 4 compares the predicted and measured values for an 80 mm cross laminated timber panel measuring 4.18 by 2.89 m mounted between two reverberation rooms. Its radiated power was measured in a reverberation room and its average velocity was measured using a laser velocimeter. Its damping loss factor was measured using structural reverberation time. It was excited with a diffuse sound field for σ_a and with a shaker for σ_p. There were two measurements of σ_p [10].

Again there was very rough agreement between theory and experiment. This is a reasonable outcome since the two measurements of σ_p were only in rough agreement with each other. This is a wall with a lower critical frequency than the first three walls considered in this paper.

Figures 5 to 7 compare theory and experimental for two 13 mm gypsum plaster board double leaf cavity wood stud walls mounted at right angles to each other to form an L shape [10]. The 50 by 100 mm wood studs were placed at 600 mm centres. There was no sound absorbing material in the wall cavities except for the case of the right hand graph in fig 7. The measurement techniques were the same as those described for figs. 1 to 3. To obtain reasonable agreement, the width of the wall in the theoretical predictions was set equal to the stud spacing following Maidanik's recommendation for ribbed panels Figure 5 shows $\boldsymbol{\sigma_a}$ and $\boldsymbol{\sigma_p}$ for a wall measuring 4.681 by 2.4 m mounted between a reverberation room and a semi-anechoic room. This wall was excited by a diffuse sound field and a shaker. Because the studs and screws can be regarded as point and line connections between the two wall leaves, the theoretical values of $\boldsymbol{\sigma_r}$, $\boldsymbol{\sigma_p}$ and $\boldsymbol{\sigma_l}$ are plotted. The theoretical value of $\boldsymbol{\sigma_l}$ gives the best agreement with the experimental value of $\boldsymbol{\sigma_r}$.

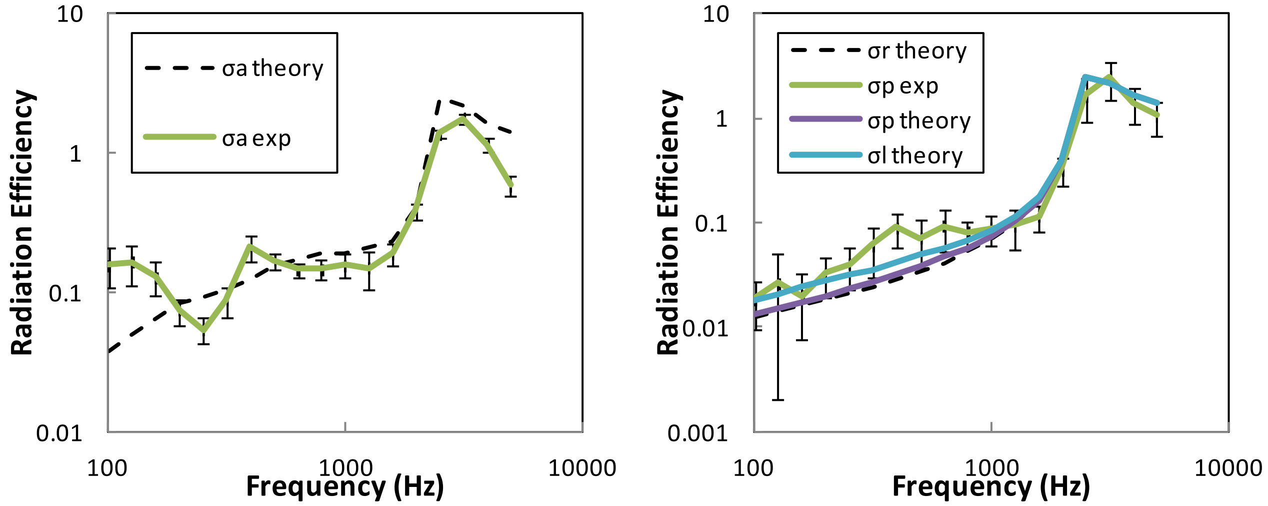

Figure 5. Comparison of the theoretical and experimental diffuse field excited (σ_a) and point excited radiation (σ_p) efficiencies for a 13 mm gypsum plaster board double leaf cavity wood stud wall measuring 4.681 by 2.4 m. The 50 by 100 mm wood studs were placed at 600 mm centres. There was no sound absorbing material in the wall cavity. The experimental 95% confidence limits are shown.

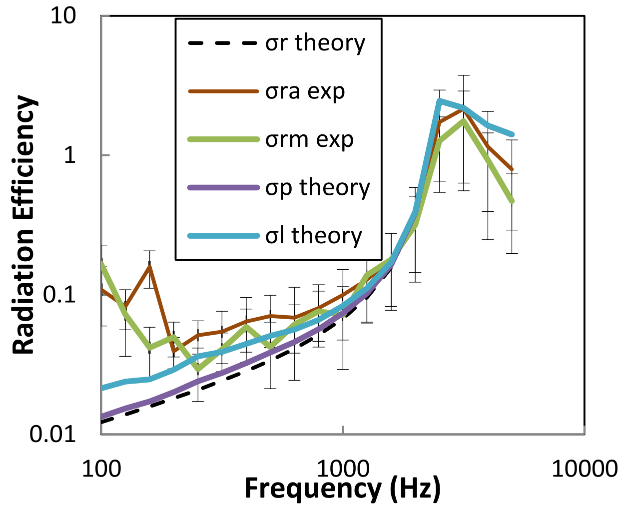

Figure 6. Comparison of the theoretical and experimental resonant radiation efficiencies for a 13 mm gypsum plaster board double leaf cavity wood stud wall measuring 3.343 by 2.353 m. The 50 by 100 mm wood studs were placed at 600 mm centres. There was no sound absorbing material in the wall cavity. The wall was excited via its right angled connection with another wall. This other wall was excited by a diffuse sound field (σ_{ra}) and mechanically by a transverse point force (σ_{rm}). The experimental 95% confidence limits are shown.

Figure 6 shows $\boldsymbol{\sigma_r}$ for the wall measuring 3.343 by 2.353 m mounted in the semi-anechoic room and connected at right angles to the previous wall. It was excited by the first wall which was excited by a diffuse sound field **($\boldsymbol{\sigma_{ra}}$)** or by a shaker **($\boldsymbol{\sigma_{rm}}$)**. Again the theoretical value of $\boldsymbol{\sigma_l}$ gives the best agreement with the experimental value of $\boldsymbol{\sigma_r}$.

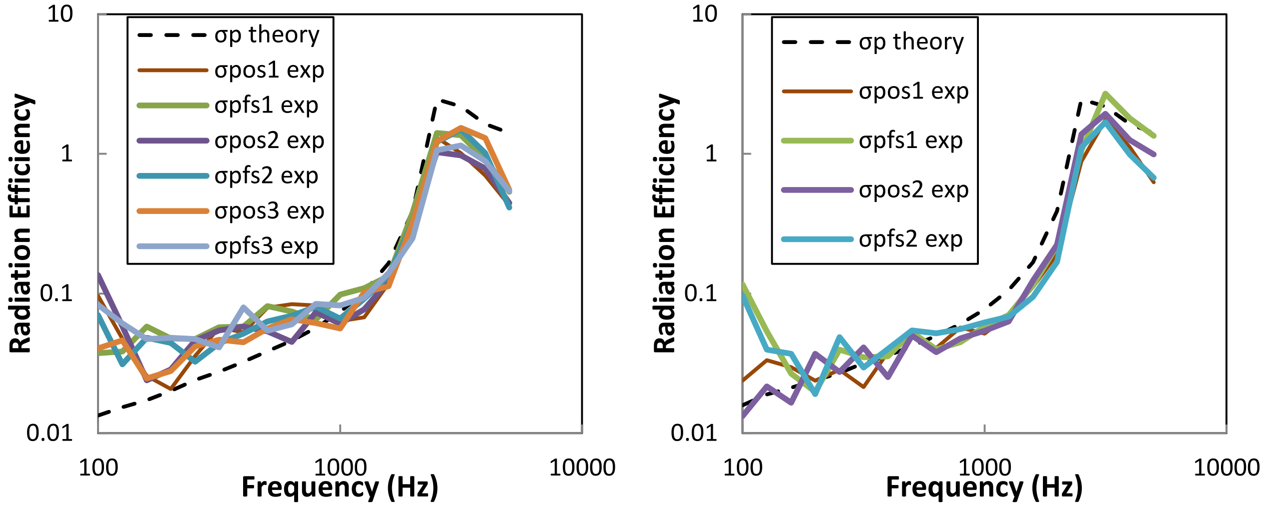

Figure 7. Comparison of the theoretical and experimental point excited radiation efficiencies (σ_p) for a 13 mm gypsum plaster board double leaf cavity wood stud wall measuring 3.343 by 2.353 m. The 50 by 100 mm wood studs were placed at 600 mm centres. There was no sound absorbing material in the wall cavity for the results in the left hand graph while there was sound absorbing material in the wall cavity for the results shown in the right hand graph.

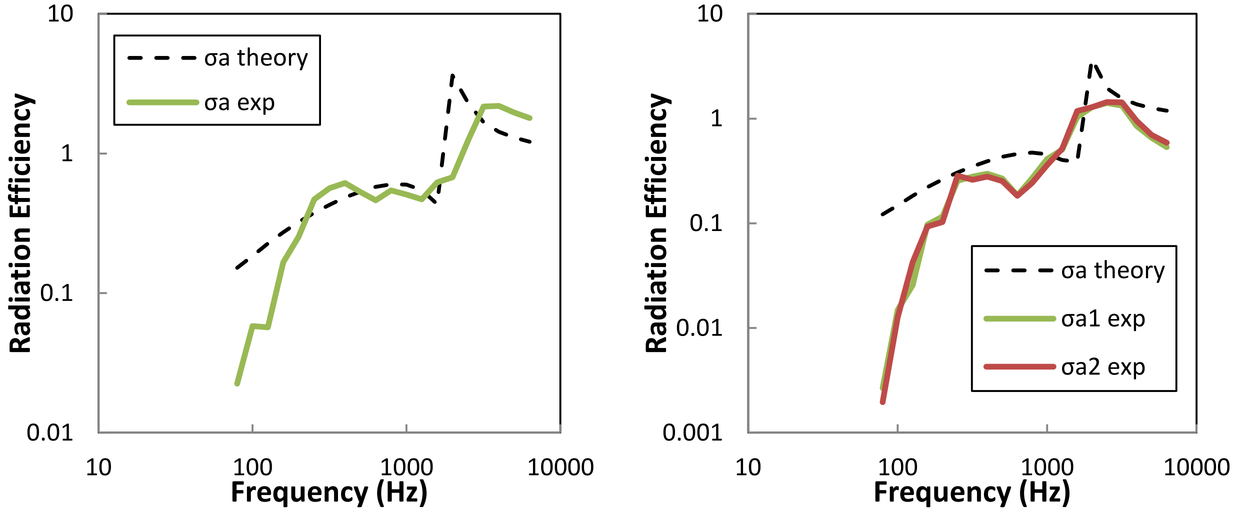

Figure 8. Comparison of the theoretical and experimental diffuse field excited (σ_a) efficiencies for a 7 mm thick plywood panel and a 12 mm thick plywood panel measuring 4.8 by 2.4 m.

Figure 7 shows the case of the second cavity wall when it was excited at different positions by a shaker. The notations "pos" and "pfs" denote excitation on and off a stud location respectively. There is little difference between on stud and off stud excitation except at low frequencies. The agreement between theory and experiment is better with these larger and more complex walls and is better for wall with sound absorbing material in its wall cavity.

Figure 8 shows the comparison between theory and experiment for σ_a for 7 mm and 12 mm plywood panels measuring 4.8 by 2.4 m. The Young's modulus and damping loss factor were determined by measurements on beams. The Young's modulus was the geometric mean of the values in the cross ply and parallel ply directions. The measurement technique was the same as that used to obtain figs 1 to 3. Again there is rough agreement between theory and experiment except at the lowest frequencies.

4. Conclusions

This paper has presented approximate formulae for predicting the resonant, point excited, line excited and diffuse field excited radiation efficiencies for a simply supported rectangular panel mounted in an infinite baffle. These formulae have been compared with the measured radiation efficiencies of single leaf panels and double leaf cavity stud walls. There is rough agreement between the theoretical formulae and the experimental measurements. Surprisingly the agreement is slightly better for the more complicated double leaf cavity stud walls.

For cavity stud walls it is necessary to use the stud spacing rather the wall length when predicting the theoretical radiation efficiency.

The damping loss factor is an important parameter because it defines the relative amplitude of the resonant and non-resonant radiation vibrational fields in the radiating wall panels. The resonant radiation efficiency depends on the boundary conditions of the radiating wall panel below the critical frequency. The exact boundary conditions of the wall panels are not normally known very accurately. Increased damping below the critical frequency will decrease the relative amplitude of the resonant vibration field and thus would be expected to increase the accuracy of the theoretical predictions.

REFERENCES

1 ISO 15712-1:2005(E) Building acoustics - Estimation of acoustic performance of buildings from the performance of elements - Part 1: Airborne sound insulation between rooms, International Organization for Standardization, Geneva, Switzerland, (2005).
2 ISO 15712-2:2005(E) Building acoustics - Estimation of acoustic performance of buildings from the performance of elements - Part 2: Impact sound insulation between rooms, International Organization for Standardization, Geneva, Switzerland, (2005).
3 Davy, J. L. The forced radiation efficiency of finite size flat panels which are excited by incident sound, *Journal of the Acoustical Society of America*, **126** (2), 694-702, (2009).
4 Leppington, F. G., Broadbent, E. G. and Heron, K. H. The acoustic radiation efficiency of rectangular panels, *Proceedings of the Royal Society of London Series a-Mathematical Physical and Engineering Sciences*, **382**, 245-271, (1982).
5 Maidanik, G. Response of ribbed panels to reverberant acoustic fields, *Journal of the Acoustical Society of America*, **34** (6), 809-826, (1962).
6 Crocker, M. J. and Price, A. J. Sound transmission using statistical energy analysis, *Journal of Sound and Vibration*, **9** (3), 469-486, (1969).
7 Davy, J. L. Sound transmission of cavity walls due to structure borne transmission via point and line connections, *Journal of the Acoustical Society of America*, **132** (2), 814-821, (2012).
8 Davy, J. L., Mahn, J. P., Guigou-Carter, C. and Villot, M. The prediction of flanking sound transmission below the critical frequency, *Journal of the Acoustical Society of America*, **132** (4), 2359-2370, (2012).
9 Mahn, J. P. and Pearse, J. R. The calculation of the resonant sound reduction index for use in EN12354, *Euronoise 2012*, Prague, Czech Republic, 10-13 June, (2012).
10 Krajči, L., Hopkins, C., Davy, J. L. and Tröbs, H.-M. Airborne sound transmission of a cross-laminated timber plate with orthotropic stiffness, *Euronoise 2012*, Prague, Czech Republic, 10-13 June, (2012).